national accelerator laboratory

EXP-15
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ACCELERATOR EXPERIMENT--Main Ring Beam Widths

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Measurement:

The main ring beam positions x and y, and full rms widths $W_x$ and $W_y$ are observed using Mike Shea's ion-profile monitor located near the downstream end of medium-straight F ($\beta_x \cong \beta_y \cong 53m$).

The results are:

t = time after injection

$$r = \frac{\text{total profile-monitor current}}{\text{beam current}}$$

$$I = \frac{\text{total profile-monitor current}}{\text{central (peak) strip current}}$$

rf on at injection

Parabola of magnet ramp: 150 msec - 350 msec

Magnet ramp: 200 GeV

Horizontal

| | t (msec) | r (Arbitrary unit) | x (in) (Arbitrary zero) | $W_x$ (in) | I (Arbitrary unit) |
|---|---|---|---|---|---|
| porch | 10 | 0.96 | 0.10 | 0.49 | 0.54 |
| | 30 | 0.97 | -0.02 | 0.40 | 0.39 |
| | 60 | 0.79 | -0.01 | 0.30 | 0.33 |
| | 100 | 0.63 | 0.13 | 0.33 | 0.36 |
| | 110 | 0.94 | 0.05 | 0.25 | 0.25 |
| parabola | 205 | 0.88 | -0.06 | 0.30 | 0.29 |
| | 250 | 0.91 | -0.21 | 0.33 | 0.33 |
| | 350 | 0.92 | -0.34 | 0.30 | 0.30 |

Vertical

| | t (msec) | r (Arbitrary unit) | y (in) (Arbitrary zero) | $W_y$ (in) | I (Arbitrary unit) |
|---|---|---|---|---|---|
| porch | 30 | 0.64 | 0.00 | 0.42 | 0.45 |
| | 50 | 0.44 | 0.02 | 0.48 | 0.49 |
| | 100 | 0.48 | -0.01 | 0.47 | 0.48 |
| | 150 | 0.63 | 0.03 | 0.41 | 0.40 |
| parabola | 200 | 0.63 | 0.04 | 0.44 | 0.44 |
| | 250 | 0.62 | 0.05 | 0.42 | 0.41 |
| | 300 | 0.63 | 0.14 | 0.38 | 0.37 |
| | 350 | 0.62 | 0.17 | 0.37 | 0.38 |
| | 450 | 0.65 | 0.15 | 0.30 | 0.30 |

Observations:

1. The values of r give a calibration of the total profile-monitor current against the beam current. Except for a few entries (Horizontal t = 60 msec and 100 msec, Vertical t = 50 msec and 100 msec) the constancy of r shows that the profile-monitor current is a good measure of the beam current.

2. The values of x and y shows that as the beam is accelerated it moves inward and upward until at the end of the parabola ramp it has moved inward by 0.34" and upward by 0.17" at the location of the profile monitor. The measurement at 70 GeV shows the closed orbit to be inward by 0.6" and downward by 0.1" at F17. This would indicate that the injection orbit is steered roughly on center horizontally but roughly 0.4" below center vertically.

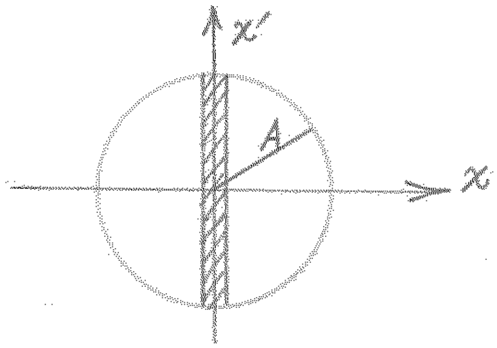
3. The beam horizontal rms width decreases on the porch from ~0.5" at injection to ~0.3". On the parabola the beam width remains roughly constant at ~0.3".

4. The beam vertical rms height remains roughly constant on the porch at ~0.45" and decreases on the parabola to ~0.3" at the end of the parabola.

5. In both planes the values of I follow the beam widths $W_x$ and $W_y$ quite well.

Interpretation and Analysis:

The fact (item 5 above) that the values of I follow the beam widths is consistent with the assumption that the beam current depends only on the maximum oscillation <u>amplitude</u> in the beam. In the normalized Floquet phase plane (designated as x,x') the beam fills up a circular area as shown. The total profile-monitor




current given by the area of the circle is proportional to $A^2$ where A is the radius of the circle or the maximum oscillation amplitude or the beam width. The central strip current given by the shaded band is proportional to A. Hence

$$I = \frac{\text{total profile-monitor current}}{\text{central strip current}} \propto \frac{A^2}{A} = A$$

and would follow the beam width.

The most prominent feature of items 3 and 4 above is that on the porch particles with large horizontal amplitude are lost slowly while the maximum vertical amplitude remains roughly constant. This indicates immediately that the particles are not lost by gas scattering which affects particles with large and small amplitudes equally and which tends to broaden the beam rather than making it narrower. The only consistent loss mechanism is that large horizontal oscillations are coupled over to the vertical plane by the $\nu_x = \nu_y$ resonance and beam is lost on the vertical aperture limit. The no-more-loss beam rms widths seem to be $W_x \cong W_y \cong 0.3"$. If we take

$2W_x \cong 2W_y \cong 0.6"$ as the maximum beam sizes, when the x-amplitude is totally coupled over to y the vertical beam size will be $0.6" \times \sqrt{2} = 0.85"$, which is a reasonable value for the present vertical aperture limit, taking into account the vertical closed-orbit distortions and bending-magnet misalignments. This also gives $\frac{0.85"}{2} = 0.43"$ for $W_y$ on the porch when the beam is continually being lost on the vertical aperture limit. Immediately after injection when the maximum beam sizes are $2W_x \cong 2W_y \cong 0.85"$ if the x-amplitude is totally coupled over to y the vertical beam size of $0.85" \times \sqrt{2} = 1.2"$ is too large for the vertical aperture limit.

To investigate more closely the coupling mechanism we start with the coupled equations

$$\begin{cases} x'' + \nu_x^2 x = Cy \\ y'' + \nu_y^2 y = Cx \end{cases} \qquad \begin{aligned} &\text{prime} = \frac{d}{d\theta} \\ &C = \frac{R^2}{B\rho} \left\langle \frac{\partial B_x}{\partial x} \right\rangle = \text{skew quadrupole field.} \end{aligned}$$

The solution which gives $x = 1$, $x' = 0$ and $y = y' = 0$ at $\theta = 0$ is

$$\begin{cases} x = \dfrac{1 + a^2 e^{-i(\nu_u - \nu_v)\theta}}{1 + a^2} e^{i\nu_u\theta} \equiv A_x e^{i\nu_u\theta} \\[2ex] y = \dfrac{a}{1 + a^2} \left[ 1 - e^{i(\nu_u - \nu_v)\theta} \right] e^{i\nu_v\theta} \equiv A_y e^{i\nu_v\theta} \end{cases}$$

where

$$|A_x|^2 + |A_y|^2 = 1$$

$$\begin{cases} \nu_u^2 = \nu_x^2 + aC \\ \nu_v^2 = \nu_y^2 - aC \end{cases} \qquad \begin{matrix} \nu_u^2 + \nu_v^2 = \nu_x^2 + \nu_y^2 \\ \nu_u^2 - \nu_v^2 = \left(\nu_x^2 - \nu_y^2\right) + 2aC \end{matrix}$$

$$a = \sqrt{1+\xi^2} - \xi, \qquad \xi = \frac{\nu_x^2 - \nu_y^2}{2C} .$$

At $\theta = 0$, $A_x = 1$, $A_y = 0$; and

$$\begin{cases} A_x = A_{x\ min} = \dfrac{1 - a^2}{1 + a^2} = \dfrac{\xi}{\sqrt{1 + \xi^2}} \\ A_y = A_{y\ max} = \dfrac{2a}{1 + a^2} = \dfrac{1}{\sqrt{1 - \xi^2}} \end{cases}$$

at $(\nu_u - \nu_v)\theta = \pi$ or at

$$\text{turn number } n = \frac{\theta}{2\pi} = \frac{1}{2(\nu_u - \nu_v)} .$$

For $\nu_x = \nu + \frac{\delta}{2}$, $\nu_y = \nu - \frac{\delta}{2}$ and with the approximation

$$\frac{\delta}{\nu} << 1, \qquad \frac{C}{\nu^2} << 1$$

we have

$$\begin{cases} \nu_u \cong \nu + \frac{1}{2}\left(\delta + \frac{aC}{\nu}\right) \\ \nu_v \cong \nu - \frac{1}{2}\left(\delta + \frac{aC}{\nu}\right) \end{cases}$$

$$n \cong \frac{1}{2\left(\delta + \frac{aC}{\nu}\right)}, \qquad \xi \cong \frac{\nu\delta}{C}, \qquad a = \sqrt{1+\xi^2} - \xi .$$

If further $\delta = 0$ (exactly on resonance)

$$\xi = 0,\ a = 1 \qquad \begin{cases} \nu_u = \nu + \dfrac{C}{2\nu} \\ \nu_v = \nu - \dfrac{C}{2\nu} \end{cases}$$

$$\begin{cases} A_{x\ min} = 0 \\ A_{y\ max} = 1 \end{cases} \quad \text{at} \quad n = \frac{\nu}{2C}$$

i.e. the x-oscillation is totally coupled over to y in $n = \frac{\nu}{2C}$ turns.

In the main ring there are two possible sources for the coupling constant C.

1. <u>Roll error of the main quadrupoles</u>

   For one 7' quad ($\ell = 2.1336$ m) with a roll error of $\alpha = 1$ mrad we have

$$C = \frac{R^2}{B\rho}\left\langle\frac{\partial B_x}{\partial x}\right\rangle = \frac{2\alpha R^2}{B\rho}\left\langle\frac{\partial B_y}{\partial x}\right\rangle = \frac{2\alpha R^2}{B\rho}\left(\frac{\partial B_y}{\partial x}\right)\frac{\ell}{2\pi R} = 0.013.$$

   However, we expect that the skew-quadrupole field due to roll errors of all quadrupoles will average to zero.

2. <u>Unbalanced bending magnet end</u>

   The inner-coil end of a bending magnet produces a skew-quadrupole field. The measured value is for each end

$$C \cong 0.01.$$

   Thus, when we are sitting exactly on resonance ($\nu_x = \nu_y$) the x-oscillation will be totally coupled over to y by a single unbalance bending magnet end in $n = \frac{\nu}{2C} \cong 1000$ turn or in ~21 msec.

This is a little too fast. It is more likely that the resonance $\nu_x = \nu_y$ is crossed back-and-forth by some mechanism. Candidates are

a. Quadrupole ripple--Ideally, the quadrupole ripple affects $\nu_x$ and $\nu_y$ in the same manner so that $\delta = \nu_x - \nu_y$ is not varied. But deviations from ideal may cause $\delta$ to wobble with the quadrupole ripple, thereby crossing $\delta = 0$.

b. Phase oscillation--If the momentum excursion is $\frac{\Delta p}{p} = 10^{-3}$ and if the remanent sextupole field in the bending magnets is not compensated we get an excursion in $\delta = \nu_x - \nu_y$ of $\sim 200 \frac{\Delta p}{p} = 0.2$. Even if the remanent sextupole field is partially compensated it is still possible that the tunes are wobbled by the phase oscillation to cross $\delta = 0$.

Suggested Checks and Experiments:

1. Similar ion-profile monitor measurements made at various times <u>on the same pulse</u> would produce much cleaner and clearer evidence. It would be preferable to lengthen the front porch to follow the couple-over over a longer period of time.

2. If the resonance $\nu_x = \nu_y$ is indeed crossed by phase oscillation, turning off the rf on the porch should reduce the couple-over and hence, reduce the beam-loss rate.

3. Trim sextupoles should affect the excursion in $\delta$ due to phase oscillation, thereby affecting the beam-loss rate on the porch.

A good measurement of the dependences of $\nu_x$ and $\nu_y$ on $\frac{\Delta p}{p}$ with given settings of the trim-sextupoles would be very useful.

4. With a given wobble of $\delta$, either the trim-quadrupoles or the tune splitter should shift the central value of $\delta$. This may be used to shift the entire range of excursion of $\delta$ away from 0.

5. Ultimately, we should reduce the coupling constant C by replacing all improper bending magnets in the ring to pair off all up and down coil ends.

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